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BOOK REVIEWS

Wave Flow of Liquid Films. By S. V. Alekseenko, V. E. Nakoryakov & B. G. Pokusaev. Begell, 1994. 313 pp. ISBN 1567800 0215. $135.

Despite the numerous applications of thin films many important problems remain unsolved, particularly those associated with the mechanisms of wave formation and their influence on heat and mass transfer. This provides motivation for the present book which is primarily concerned with nonlinear interfacial waves on thin liquid films moving under gravity and is based, to a large extent, on the works of the authors between 1970 and 1990 at the Institute of Thermophysics, Novosibirsk. Research in the area of falling liquid films started with the pioneering experimental work by Kapitza & Kapitza (1949). A thin liquid film falling down an inclined plane under the action of gravity provides a simple example of an open flow hydrodynamic instability starting with an initial disturbance of specific wavenumber and leading through a series of secondary nonlinear transitions to a rich spatio-temporal wave dynamics.

Two early chapters give details of experimental set-ups for studying thin film flows on the external surface of a vertical cylinder and down an inclined plane. They also include an extensive range of measurement techniques for local film thickness, velocity and wall shear stress.

The main theoretical analysis is in chapter 6 'Wave Motion Modelling', starting with a review of classical wave mechanisms and wave equations. The long wave approximation is used to derive equations for two- and three-dimensional weakly nonlinear waves for Reynolds numbers of order unity. For 'moderate' Reynolds numbers the boundary layer equations are obtained and integrated using the Kármán–Pohlhausen method. It is shown that disturbances propagating on liquid films flowing down an incline are in general the result of a nonlinear interaction between kinematic, inertial, capillary and gravitational waves. For example kinematic waves acquire energy from inertial waves and transmit it to capillary waves.

A detailed description of two-dimensional periodic stationary waves is given in chapter 8. For natural waves – arising from the growth of linear disturbances and the emergence of nonlinear effects – the problem of wave selection, from a range of possible solutions, is addressed. In the case of waves excited by external forcing, families of spatially periodic solutions arising from the bifurcation analysis of Chang (1986, 1989), Demekhin & Shkadov (1986), Tsvelodub (1988, 1990) are described and compared with experiment.

Chapters 9 and 10 consider the evolution of two-dimensional solitary perturbations and three-dimensional waves again from both a theoretical and an experimental standpoint. Finally chapter 13 is reserved for an investigation of the effect of this range of nonlinear waves on the processes of heat and mass transfer at solid boundaries and interfaces.

The reader is left admiring a presentation of the significant work in the field up to 1990 in a way which carefully balances experiment with mathematical modelling and analysis. More recent developments in the area include construction and stability analysis of all families of stationary waves (Chang, Demekhin & Kopelevich 1993), experimental investigations of sideband and subharmonic instabilities (Lin & Gollub 1993) and theoretical investigations of the nonlinear wave selection mechanisms and transition to irregular spatio-temporal behaviour (Chang 1994; Chang & Demekhin 1996).

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M. D. Savage

Numerical Solution of Partial Differential Equations. By K. W. Morton & D. F. Mayers. Cambridge University Press, 1994. 227 pp. £13.95.

In this short but illuminating monograph, the authors present an introduction to finite-difference methods for solving first- and second-order partial differential equations, with emphasis on three pervasive equations of mathematical physics: the unsteady heat conduction equation; the convection equation; and the Poisson equation. The book is written from the perspective of the applied mathematician, but most of it should be accessible to researchers and students in engineering and applied physical sciences with a rudimentary background in numerical methods and computer programming, and a graduate-level knowledge of mathematics. The depth and breadth of the discussion have been optimized to fit a sixteen-lecture advanced undergraduate or beginning graduate course in applied mathematics. Bibliographic notes and a collection of unsolved problems are given at the end of each chapter.

The titles of the seven chapters are: Introduction; Parabolic equations in one space variable; Parabolic equations in two and three dimensions; Hyperbolic equations in one space dimension; Consistency, convergence, and stability; Linear second-order elliptic equations in two dimensions; Iterative solution of linear algebraic equations. Chapter 5 includes a short section on variational formulations and the finite-element method. The chapter contents are distillates of those found in full blown monographs on finite-difference methods such as that by Richtmyer and the first author.

This book is an attractive text for use with a course on its subject; an alternative would be a more general text on numerical methods that includes finite-difference methods as a subtopic. The vantage point of this book is the successful fusion of mathematical concepts and practical advice. Engineers will find the rigorous style refreshing, and applied mathematicians will find the practical perspective reassuring.

C. Pozrikidis